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Quadratic Equations

Introduction

Any equation whose degree is 2 can be termed as a quadratic equation.

Its general form is represented as $ax^2 + bx + c = 0$, where a, b and c are real numbers with $a \neq 0$.

Example:

$x^2 + 5x + 6 = 0$

$x^2 - 16 = 0$

$\sqrt{2}x^2 + 7x = 0$

Roots of an equation

Roots of an equation are values of x at which the quadratic equation satisfies the general form.

In other words, if ‘k’ is a root of $f(x) = ax^2 + bx + c$, then it means that $f(k) = 0$

Also, $(x – k)$ will be a factor of the above equation.

Finding roots/solutions

There are 2 main ways of finding the solutions of an equation.

(i) Middle term splitting/factorization.
(ii) Finding the discriminant.

Middle Term Splitting/Factorization

This is the most basic and efficient method of finding roots.
If $f(x) = ax^2 + bx + c$
b – represents the sum of roots,
c – represents the product of roots

For example: $f(x) = x^2 + 5x + 6$
$= x^2 + 3x + 2x + 6$
$= x(x + 3) + 2(x + 3)$
$= (x + 2)(x + 3)$
Roots are -2 and -3.

$F(x) = 3x^2 - 4x – 7$
$= 3x^2 + 3x – 7x – 7$
$= 3x(x + 1) - 7(x + 1) = (3x - 7)(x + 1)$
Roots are 7/3, -1

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Discriminant Method (Sridharacharya Method)

This is a universal method and can be used for any quadratic equation.

To find the roots of an equation $ax^2 + bx + c$, we find out its discriminant. It is represented by D.

And it can be calculated by the equation $D = b^2 - 4ac$

Once we have the discriminant, we can directly find out the roots by putting values.

$x = \frac{-b \pm \sqrt{D}}{2a}$ or in other words $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Nature of Roots

Using the Discriminant, we can also find the nature of the roots.

If D > 0, then roots are real and unequal.

If D = 0, then roots are real and equal.

If D < 0, then roots are unreal/imaginary and unequal.

Nature of Roots

If D is a perfect square then the roots are rational and in case it is not a perfect square then the roots are irrational and conjugate pairs.

If $a = 1$, $b , c \in I$ and the roots are rational numbers, then the root must be an integer.

If a quadratic equation in x has more than two roots, then it is an identity in x.

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Important Identities

$(x + y)^2 = x^2 + y^2 + 2xy$

$(x - y)^2 = x^2 + y^2 - 2xy$

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$x^2 - y^2 = (x + y)(x - y)$

$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Formation of Equation

If α and β are the two roots of $f(x) = ax^2 + bx + c$, we can also find the relation between the roots and a, b and c.

Basically, every quadratic equation can be expressed as
x^2 - (Sum of roots)x + (Product of roots) = 0
$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$
or, $(x - \alpha)(x - \beta) = 0$

On comparing both the forms, we can deduce that
$\alpha + \beta = -b/a$
$\alpha\beta = c/a$

If the two roots are reciprocal to each other, then a = c.
If the two roots are equal in magnitude and opposite in sign, then b = 0.

Comparing Quadratic Equations

In each of these questions, two equations A and B are given. You have to solve both the equations and give answer:

a) If $x < y$
b) If $x > y$
c) If $x \geq y$
d) If $x \leq y$
e) If no relationship can be established between x and y.

Sign of roots

Sign of (α + β)	Sign of (αβ)	Sign of α	Sign of β
+	+	+	+
–	+	–	–
+	–	+	–
–	–	–	+

Note: α is greater than β in absolute terms (ignoring the sign)

Practice Questions

Q.1) In each of these questions, two equations A and B are given. You have to solve both the equations and give answer:

A: $x^2 + 13x + 40 = 0$
B: $y^2 + 14y - 51 = 0$

a) If $x < y$
b) If $x > y$
c) If $x \geq y$
d) If $x \leq y$
e) $x = y$ or no relationship can be established

Practice Questions

Answer – Option E

Practice Questions

Q.2) In each of these questions, two equations A and B are given. You have to solve both the equations and give answer:

A: $3x^2 + 8x + 4 = 0$
B: $4y^2 - 19y + 12 = 0$

a) If $x < y$
b) If $x > y$
c) If $x \geq y$
d) If $x \leq y$
e) $x = y$ or no relationship can be established

Practice Questions

Answer – Option A

Practice Questions

Q.3) In each of these questions, two equations A and B are given. You have to solve both the equations and give answer:

A: $18x^2 + 31x + 6 = 0$
B: $3y^2 + 19y + 28 = 0$

a) If $x < y$
b) If $x > y$
c) If $x \geq y$
d) If $x \leq y$
e) $x = y$ or no relationship can be established

Practice Questions

Answer – Option B

Practice Questions

Q.4) In each of these questions, two equations A and B are given. You have to solve both the equations and give answer:

A: $7x^2 - 13x - 24 = 0$
B: $6y^2 + 19y + 14 = 0$

a) If $x < y$
b) If $x > y$
c) If $x \geq y$
d) If $x \leq y$
e) $x = y$ or no relationship can be established

Practice Questions

Answer – Option B

Practice Questions

Q.5) Find the roots of the following expression.

$((x + 8)/(x - 8)) - ((x - 8)/(x + 8)) = 8/3$

a) 4, 16
b) -4, 16
c) 4, -8
d) -4, -8

Practice Questions

Answer – Option B

Practice Questions

Q.6) Find the value of K for which x = 2 is a root of the equation $3x^2 - 2Kx + 5 = 0$.

a) 17/4
b) 13/4
c) 15/4
d) 19/4
e) 11/4

Practice Questions

Answer – Option A

Practice Questions

Q.7) Find the quadratic equation whose roots are -3, 5.

a) $x^2 - 8x - 15 = 0$
b) $x^2 + 8x - 15 = 0$
c) $x^2 - 2x + 15 = 0$
d) $x^2 + 2x - 15 = 0$
e) $x^2 - 2x - 15 = 0$

Practice Questions

Answer – Option E

Practice Questions

Q.8) If α, β are the roots of the equation $2x^2 - 3x + 2 = 0$, form the equation whose roots are α^2, β^2.

a) $4x^2 - x - 4 = 0$
b) $4x^2 + x + 4 = 0$
c) $4x^2 - x + 4 = 0$
d) $4x^2 + x - 4 = 0$
e) $x^2 - x + 4 = 0$

Practice Questions

Answer – Option C

Practice Questions

Q.9) Determine k for which the roots of the equation $9x^2 + 2kx + 4 = 0$ are equal.

a) ± 4
b) ± 7
c) ± 5
d) ± 6
e) ± 8

Practice Questions

Answer – Option D

Practice Questions

Q.10) Find the range of x which will satisfy the below inequality:

$- x^2 + 6x - 8 > 0$

a) $-2 < x < 2$
b) $2 < x < 4$
c) $-2 < x < 4$
d) $-4 < x < -2$
e) $-4 < x < 2$

Practice Questions

Answer – Option B

Practice Questions

Q.11) The value of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2 are:

a) ± 4
b) ± 2
c) ± 10
d) ± 6
e) ± 8

Practice Questions

Answer – Option D

Practice Questions

Q.12) i) a and b are the roots of the quadratic equation $2x^2 - jx + k = 0$, where $(a/4) + (b/2) = 16$ and the value of k = (√15625) x (625)^(1/4) − 1.

ii) The value of a and b must satisfy the condition $15 > (b - a) > 6$

Find which of the following quadratic equations has roots (a/2, b/2)

a) $2x^2 - 19x + 39 = 0$
b) $x^2 - 18x + 72 = 0$
c) $x^2 + 7x - 78 = 0$
d) $x^2 - 41x + 78 = 0$
e) None of these

Practice Questions

Answer – Option E

Practice Questions

Q.13) Equation (1): $(x^2/5) - (a + 25)x/5 + 5a = 0$
Equation (2): $(y^2/2) + 25b = (2b + 25)y/2$

Note: A quadratic equation $z^2 - 52z + 576 = 0$, the smaller root of this quadratic is one of the roots of Eq. (1) and the larger root of this equation is one of the roots of Eq. (2).

Which of the following is/are true?
(I) Larger root of Equation (1) is 5^2
(II) Product of the Larger roots of both eq.(1) and (2) is 900
(III) $(a + b) = 36$

a) Only I and III are true
b) Only II and III are true
c) Only I is true
d) Only I and II are true
e) All I, II and III are true

Practice Questions

Answer – Option D

Practice Questions

Q.14) P, R and T are the consecutive whole numbers but not in same order, and P + T + R = 3 and also P > T < R and (P/R) is a natural number

Equation: $x^P/4 - (19/2)\, x^R + 88x^T = 0$

then find the product of the larger root of the quadratic equation and the sum of the digits of the smaller root?

a) 176
b) 132
c) 165
d) 142
e) 154

Practice Questions

Answer – Option E

Practice Questions

Q.15) Eq. 1: $(1/2) + 14x^{-2} = 5.75x^{-1}$
Eq. 2: $0.125y^2 – 0.4375y = 1.875$

If the smaller root of equation 'II' and larger root of equation 'I' are used to form another equation 'C' in the form of variable 'k', then then which of the represents the equation 'W'?

a) $2k^2 – 11k = 40$
b) $2k^2 – 10k = 40$
c) $2k^2 – 11k = 30$
d) $k^2 – 11k = 40$
e) $k^2 – 10k = 30$

Practice Questions

Answer – Option A

Practice Questions

Q.16) The following expressions form a series followed by a (?). Find the odd one out first and then find that what will come in place of the question mark (?) according to the corrected series?

(x - 10), (x + 10), 120, x(x - 10), 4500, (1540 × x), ?

Also, 'x' = positive root of $x^2 - 6x - 720 = 0$

a) (20020 × x)
b) (20120 × x)
c) (20320 × x)
d) (20420 × x)
e) (20220 × x)

Practice Questions

Answer – Option A

Practice Questions

Q.17) A: $(a \times a) - 3a - \sqrt{(4a^2)} = -6$
B: $(b^2) - \sqrt{(81b^2)} = -4 \times (5)$
C: $(c^2 \sqrt{625c^6}) / 5c^3 + (4 \times 7) = 39c$
D: $d^2 - (3 \times 5)d = (7 \times (-8))$

Find the LCM of all the larger roots of equations A, B, C, D.

a) 440
b) 840
c) 740
d) 860
e) 720

Practice Questions

Answer – Option B

Practice Questions

Q.18) A: $(a \times a) - 3a - \sqrt{(4a^2)} = -6$
B: $(b^2) - \sqrt{(81b^2)} = -4 \times (5)$
C: $(c^2 \sqrt{625c^6}) / 5c^3 + (4 \times 7) = 39c$
D: $d^2 - (3 \times 5)d = (7 \times (-8))$

Find the difference between the larger and smaller roots of equation A.

a) -1
b) 2
c) 1
d) 3
e) -2

Practice Questions

Answer – Option C

Practice Questions

Q.19) A: $(a \times a) - 3a - \sqrt{(4a^2)} = -6$
B: $(b^2) - \sqrt{(81b^2)} = -4 \times (5)$
C: $(c^2 \sqrt{625c^6}) / 5c^3 + (4 \times 7) = 39c$
D: $d^2 - (3 \times 5)d = (7 \times (-8))$

In which of the following equations difference between larger and smaller root is 1.

a) Only A and B
b) Only B
c) Only B and C
d) Only A, B and D
e) None of these

Practice Questions

Answer – Option D

Practice Questions

Q.20) Quantity I: If one root of the equation $x^2 + kx - 8 = 0$ is square root of the other root, then find the value of 'k'. (where 'x' and 'k' are real numbers)

Quantity II: $4x^2 - 11x + 2k = 0$ and $x^2 - 3x - k = 0$ have a common root. What is the value of k?

[a] Quantity I ≥ Quantity II
[b] Quantity I > Quantity II
[c] Quantity I < Quantity II
[d] Quantity I ≤ Quantity II
[e] Quantity I = Quantity II or no relation can be established between Quantity I and II

Practice Questions

Answer – Option C

Practice Questions

Q.21) Consider the below quadratic equation:
$y^2 - 60y + p = 0$

It is given that both the roots of the above equation are prime numbers. Find the number of possible values of p.

[a] 3
[b] 5
[c] 7
[d] 6
[e] 4

Practice Questions

Answer – Option D

Practice Questions

Q.22) Consider the below given two equations:
Equation 1: $24P^2 - 22P - 10 = 0$
Equation 2: $6Q^2 + 2Q - 20 = 0$

Which of the following statements is definitely false related to the above given equations:

A. Value of Q is more than that of P.
B. The difference of both roots of equation 1 and the sum of both roots of equations 2 are co-prime numbers.
C. Maximum value of P is 50% of that of Q.

[a] A and B only
[b] A, B and C
[c] B and C only
[d] Only B
[e] A and C only

Practice Questions

Answer – Option B

Practice Questions

Q.23) If one of the roots in equation III is equal to '(m + n)', find the value of 'p'.

I: $x^2 + 329 - 38x + m = (x - 19)^2$
II: $y^2 + 669 - 54y = (y - n)^2 - 60$
III: $z^2 + 34z - p = 0$

[a] 5527
[b] 5387
[c] 5467
[d] 5397
[e] 5487

Practice Questions

Answer – Option E

Practice Questions

Q.24) In a Maths test, two students, Rohit and Mohit, while solving a quadratic equation, committed the following mistakes:

(i) Rohit committed an error in the constant term and got the roots as 5 and 9.
(ii) Mohit made a mistake in the coefficient of x and got the roots as 12 and 4.

Find the correct quadratic equation.

[a] $x^2 - 16x + 45 = 0$
[b] $x^2 - 4x + 16 = 0$
[c] $x^2 - 7x + 24 = 0$
[d] $x^2 - 8x + 45 = 0$
[e] $x^2 - 14x + 48 = 0$

Practice Questions

Answer – Option E

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